

Package: topologyR (via r-universe)

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Title Topological Connectivity Analysis for Numeric Data

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Description Topological data analysis methods based on graph-theoretic approaches for discovering topological structures in data. Constructs topological spaces from graphs following Nada et al. (2018) <[doi:10.1002/mma.4726](https://doi.org/10.1002/mma.4726)>, with visibility graph construction for time series following Lacasa et al. (2008) <[doi:10.1073/pnas.0709247105](https://doi.org/10.1073/pnas.0709247105)>. Supports directed visibility graphs for bitopological analysis of temporal irreversibility (Kelly, 1963), and Alexandrov topology construction from reachability preorders.

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Contents

| | |
|---|-----------|
| analyze_topology_factors | 2 |
| bitopology_invariants | 3 |
| calculate_thresholds | 5 |
| calculate_topology | 6 |
| complete_topology | 6 |
| generate_alexandrov_topology | 7 |
| generate_bitopology | 9 |
| generate_topology | 10 |
| horizontal_visibility_graph | 12 |
| is_topology_connected | 14 |
| is_topology_connected_exact | 14 |
| is_topology_connected_manual | 15 |
| is_topology_connected2 | 16 |
| natural_visibility_graph | 16 |
| simplest_topology | 18 |
| visualize_topology_thresholds | 18 |
| Index | 20 |

analyze_topology_factors

Analyze topology characteristics for different IQR factors

Description

Analyzes how different IQR factors affect topology characteristics. Helps determine the optimal factor by showing how the factor choice impacts base size and set sizes.

Note: This function uses threshold-based neighborhoods (metric approximation), not the graph-theoretic approach of Nada et al. (2018). For the theoretically faithful approach, use visibility graphs with [generate_topology](#).

Usage

```
analyze_topology_factors(data, factors = NULL, plot = TRUE)
```

Arguments

| | |
|---------|---|
| data | Numeric vector containing the data to analyze. |
| factors | Numeric vector of factors to test (default: c(1, 2, 4, 8, 16)). |
| plot | Logical, whether to return a plot object (default: TRUE). |

Value

A data.frame with columns:

factor Numeric. The IQR factor used.

threshold Numeric. The calculated threshold (IQR/factor).

base_size Integer. Number of sets in the base.

max_set_size Integer. Size of the largest set.

min_set_size Integer. Size of the smallest set.

If `plot = TRUE`, the data.frame carries an attribute "plot" containing a ggplot object.

Examples

```
data <- rnorm(50)
results <- analyze_topology_factors(data)
print(results)
```

`bitopology_invariants` *Compute bitopological invariants from forward and backward topologies*

Description

Given the forward topology τ^+ and backward topology τ^- (as returned by `generate_topology`), computes bitopological invariants that quantify temporal irreversibility and structural asymmetry.

Usage

```
bitopology_invariants(
  forward,
  backward,
  n_elements,
  undirected = NULL,
  alexandrov = NULL
)
```

Arguments

| | |
|-------------------------|--|
| <code>forward</code> | The forward Nada topology τ^+ (output of <code>generate_topology</code> called with forward adjacency). |
| <code>backward</code> | The backward Nada topology τ^- (output of <code>generate_topology</code> called with backward adjacency). |
| <code>n_elements</code> | Integer. Total number of elements in the ground set V . |
| <code>undirected</code> | The undirected Nada topology (optional, for comparison). |
| <code>alexandrov</code> | The Alexandrov topology (optional, for resolution comparison). |

Details

Pairwise connectedness (Kelly, 1963): the bitopological space (X, τ^+, τ^-) is pairwise disconnected if there exists a proper non-empty subset $A \subset X$ that is simultaneously τ^+ -open and τ^- -closed (its complement is τ^- -open). This is checked by iterating over all τ^+ open sets and testing whether their complement is τ^- -open.

Irreversibility prediction (Lacasa & Toral, 2010): for a time series with asymmetric dynamics (e.g., gradual expansions and abrupt recessions), the forward topology τ^+ should be more connected than τ^- , because forward visibility is less obstructed during gradual rises than backward visibility after abrupt drops.

Resolution gain: the difference $|\mathcal{B}_{\text{Nada}}| - |\mathcal{B}_A|$ quantifies how much additional structure the Nada intersection/union closure captures beyond the pure Alexandrov order structure. A large gain means the Nada pipeline is extracting genuinely new topological information.

Value

A list with:

forward_components Integer. Number of connected components in τ^+ .

backward_components Integer. Number of connected components in τ^- .

forward_connected Logical. Whether τ^+ is connected.

backward_connected Logical. Whether τ^- is connected.

forward_base_size Integer. Number of base elements in τ^+ .

backward_base_size Integer. Number of base elements in τ^- .

irreversibility_components Numeric in the range 0 to 1. Normalized asymmetry of component counts: $|C^+ - C^-| / \max(C^+, C^-)$. Zero means equal component counts (necessary for reversibility).

irreversibility_base Numeric in the range 0 to 1. Normalized asymmetry of base sizes.

asymmetry_direction Integer. $C^- - C^+$. Positive means the forward topology is more connected (fewer components) than the backward topology, consistent with gradual expansions (forward visibility) and abrupt contractions (backward obstruction).

pairwise A list with pairwise connectedness results. Contains `pairwise_connected` (logical or NA), `clopen_forward` (integer vector, the forward-open / backward-closed set if disconnected), and `clopen_backward` (its complement). Requires both topologies to be fully enumerated (`max_open_sets > 0` and `complete = TRUE`).

resolution A list comparing the Nada and Alexandrov topologies (if `alexandrov` is provided). Contains base size comparisons and relative resolution gains.

undirected_info A list with undirected topology summary (if `undirected` is provided).

References

Kelly, J. C. (1963). Bitopological spaces. *Proceedings of the London Mathematical Society*, 3(1), 71-89.

See Also

[generate_bitopology](#) for the complete pipeline.

Examples

```
series <- c(3, 1, 4, 1, 5, 9, 2, 6)
g <- horizontal_visibility_graph(series, directed = TRUE)
tf <- generate_topology(g$out_adjacency, g$n, max_open_sets = 0L)
tb <- generate_topology(g$in_adjacency, g$n, max_open_sets = 0L)
inv <- bitopology_invariants(tf, tb, g$n)
inv$asymmetry_direction
```

calculate_thresholds *Calculate multiple threshold methods for topology analysis*

Description

Computes five different threshold methods for defining neighborhoods in threshold-based topology construction.

Note: Threshold-based methods produce metric topologies, not the graph-induced topologies of Nada et al. (2018). See [generate_topology](#) for the theoretically faithful approach.

Usage

```
calculate_thresholds(data)
```

Arguments

data Numeric vector to calculate thresholds for.

Value

A named list with five threshold values:

mean_diff Mean of absolute differences between adjacent sorted values.

median_diff Median of absolute differences between adjacent sorted values.

sd Standard deviation of the data.

iqr IQR divided by 4.

dbscan k-th nearest neighbor distance ($k = \text{ceiling}(\log(n))$).

Examples

```
calculate_thresholds(rnorm(100))
```

calculate_topology *Calculate topology base size for a given threshold*

Description

Calculate topology base size for a given threshold

Usage

```
calculate_topology(data, threshold)
```

Arguments

data Numeric vector to analyze.
 threshold Numeric value for the threshold parameter. Must be non-negative.

Value

integer scalar. The number of sets in the topological base.

Examples

```
calculate_topology(rnorm(30), threshold = 0.5)
```

complete_topology *Create a complete-graph topology from data*

Description

Constructs a topology from the complete graph on the data points. In the complete graph, every vertex is a neighbor of every other vertex, so each neighborhood is $N(v) = V \setminus \{v\}$. The resulting topology is generated using the fixed-point closure algorithm.

Note: The complete graph produces trivial neighborhoods (all vertices except self). For meaningful topological structure, use visibility graphs with [generate_topology](#).

Usage

```
complete_topology(data, verify_axioms = FALSE)
```

Arguments

data Numeric vector containing the data points. Length must be between 2 and 64.
 verify_axioms Logical. Whether to verify topology axioms (default: FALSE).

Value

A list with the same structure as [generate_topology](#).

Examples

```
result <- complete_topology(c(1, 2, 3, 4, 5))
result$n_open_sets
result$connected
```

```
generate_alexandrov_topology
```

Generate the Alexandrov topology from a directed visibility graph

Description

Computes the Alexandrov topology induced by a directed acyclic graph (DAG) via bitset-based reachability propagation. The open sets are the upsets of the reachability relation: vertex v 's minimal open set is the set of all vertices reachable from v via directed paths (including v itself).

The Alexandrov topology is a **lower bound** on the resolution of the Nada et al. (2018) topology: $\tau_A \subseteq \tau_{\text{Nada}}^+$. The difference in base sizes quantifies how much additional structure the intersection/union closure of Nada et al. captures beyond the pure order structure.

Usage

```
generate_alexandrov_topology(
  out_adjacency,
  n_elements,
  max_open_sets = 0L,
  check_connected = TRUE,
  verify_axioms = FALSE
)
```

Arguments

- | | |
|-----------------|---|
| out_adjacency | A list of integer vectors, where element i contains the 1-based indices of all forward (later-in-time) neighbors of node i . Must represent a DAG where all edges go from lower to higher index. Typically obtained from horizontal_visibility_graph or natural_visibility_graph with <code>directed = TRUE</code> . |
| n_elements | Integer. Total number of elements in the ground set V . |
| max_open_sets | Integer. Maximum number of open sets to enumerate (default: 0, meaning skip enumeration). The Alexandrov topology can have up to 2^n open sets, so enumeration is only feasible for small n . Connectivity is always computed from the base without enumeration. |
| check_connected | Logical. Whether to compute exact topological connectivity via the specialization preorder (default: TRUE). |
| verify_axioms | Logical. Whether to verify topology axioms after enumeration (default: FALSE). Only meaningful when the topology is fully enumerated. |

Details

The algorithm computes reachability in $O(n \cdot m / 64)$ time by processing vertices in reverse topological order (= reverse time order for visibility graphs) and propagating reachability sets as multi-word bitsets with OR operations. This reuses the same bitset infrastructure as the Nada engine with compile-time template dispatch for $n \leq 192$.

The Alexandrov topology on a finite set equipped with a preorder is canonical: there is a bijection between preorders and Alexandrov topologies (Alexandrov, 1937). For directed visibility graphs, the reachability preorder captures the temporal ordering structure of the time series.

Value

A list with the same structure as [generate_topology](#):

subbase List of integer vectors. The upsets (= base for Alexandrov).

base List of integer vectors. Same as subbase (upsets are already closed under intersection).

base_complete Always TRUE (no iteration needed).

connected Logical. Exact topological connectivity.

components List of integer vectors. Exact connected components.

topology List of integer vectors, or NULL. Open sets if enumerated.

n_open_sets Integer, or NA. Number of open sets if enumerated.

complete Logical. Whether enumeration finished.

References

Alexandrov, P. (1937). Diskrete Raume. *Matematicheskii Sbornik*, 2(44), 501-519.

See Also

[generate_topology](#) for the Nada et al. topology, [generate_bitopology](#) for the complete bitopological analysis.

Examples

```
series <- c(3, 1, 4, 1, 5)
g <- horizontal_visibility_graph(series, directed = TRUE)
alex <- generate_alexandrov_topology(g$out_adjacency, g$n)
alex$connected
length(alex$base)
```

generate_bitopology *Generate a bitopological analysis from a time series*

Description

Performs a complete bitopological analysis of a time series by constructing directed visibility graphs and computing four topological spaces:

1. **Undirected Nada topology** τ : from the symmetric (undirected) visibility graph (standard Nada et al. 2018).
2. **Forward Nada topology** τ^+ : from closed forward neighborhoods $N^+[v] = \{v\} \cup \{w : v \rightarrow w\}$.
3. **Backward Nada topology** τ^- : from closed backward neighborhoods $N^-[v] = \{v\} \cup \{w : w \rightarrow v\}$.
4. **Alexandrov topology** τ_A : from reachability upsets of the directed graph.

The pair (X, τ^+, τ^-) forms a **bitopological space** (Kelly, 1963). The divergence between τ^+ and τ^- quantifies temporal irreversibility: in a reversible process, $\tau^+ \cong \tau^-$; in an irreversible process (e.g., asymmetric business cycles), they diverge.

Usage

```
generate_bitopology(
  series,
  graph_type = c("hvg", "nvg"),
  max_open_sets = 0L,
  max_base_sets = 100000L,
  alexandrov = TRUE
)
```

Arguments

| | |
|---------------|---|
| series | Numeric vector representing the time series. |
| graph_type | Character. Either "hvg" (Horizontal Visibility Graph) or "nvg" (Natural Visibility Graph). Default: "hvg". |
| max_open_sets | Integer. Maximum open sets to enumerate per topology (default: 0, skip enumeration). Required for pairwise connectedness check. |
| max_base_sets | Integer. Maximum base elements during intersection closure for the Nada topologies (default: 100,000). |
| alexandrov | Logical. Whether to also compute the Alexandrov topology for resolution comparison (default: TRUE). |

Details

The existing undirected engine is called three times (undirected, forward, backward) and the Alexandrov engine once. No new C++ code is needed for the forward and backward topologies: passing directed adjacency lists to the existing `generate_topology` produces the correct closed neighborhoods $N^+[v]$ or $N^-[v]$ automatically (the engine always adds the self-loop $v \in N[v]$).

Value

A list with:

- graph** The directed visibility graph (output of [horizontal_visibility_graph](#) or [natural_visibility_graph](#) with `directed = TRUE`).
- undirected** The undirected Nada topology (output of [generate_topology](#)).
- forward** The forward Nada topology τ^+ .
- backward** The backward Nada topology τ^- .
- alexandrov** The Alexandrov topology τ_A (if requested).
- invariants** Bitopological invariants (output of [bitopology_invariants](#)).

References

- Kelly, J. C. (1963). Bitopological spaces. *Proceedings of the London Mathematical Society*, 3(1), 71-89.
- Lacasa, L., & Toral, R. (2010). Description of stochastic and chaotic series using visibility graphs. *Physical Review E*, 82(3), 036120.

See Also

[bitopology_invariants](#) for computing invariants from pre-computed topologies.

Examples

```
series <- c(3, 1, 4, 1, 5, 9, 2, 6)
bt <- generate_bitopology(series, graph_type = "hvg")
bt$invariants$forward_components
bt$invariants$backward_components
bt$invariants$irreversibility_components
```

| | |
|-------------------|--|
| generate_topology | <i>Generate a topology from a graph's neighborhood structure</i> |
|-------------------|--|

Description

Given a graph (represented by its adjacency list of neighborhoods), generates the induced topology following Nada et al. (2018). The procedure is:

1. The neighborhoods $N(v)$ form the **subbase**.
2. The base is the closure of the subbase under finite intersections (wavefront propagation in C++).
3. Topological connectivity is determined **exactly** via the specialization preorder on the base (Alexandrov theorem for finite spaces), without requiring topology enumeration.
4. Optionally, the full topology (closure under arbitrary unions) is enumerated, subject to a `max_open_sets` limit.

The connectivity result is **exact**: the connected components returned are the true topological connected components, not approximations.

Usage

```

generate_topology(
  adjacency,
  n_elements,
  max_open_sets = 1000000L,
  max_base_sets = 100000L,
  check_connected = TRUE,
  verify_axioms = FALSE
)

```

Arguments

| | |
|-----------------|---|
| adjacency | A list of integer vectors, where element <i>i</i> contains the 1-based indices of all neighbors of node <i>i</i> . Typically obtained from horizontal_visibility_graph or natural_visibility_graph . |
| n_elements | Integer. Total number of elements in the ground set <i>V</i> . |
| max_open_sets | Integer. Maximum number of open sets to enumerate (default: 1,000,000). If the topology exceeds this limit, enumeration stops and <code>complete = FALSE</code> . Connectivity is unaffected (computed from the base, not the topology). Set to 0 to skip enumeration entirely. |
| max_base_sets | Integer. Maximum number of base elements during intersection closure (default: 100,000). If exceeded, closure stops early and <code>base_complete = FALSE</code> . Connectivity results remain valid but may be approximate if the base is incomplete. |
| check_connected | Logical. Whether to compute exact topological connectivity via the specialization preorder (default: TRUE). |
| verify_axioms | Logical. Whether to verify topology axioms after enumeration (default: FALSE). Only meaningful when the topology is fully enumerated (<code>complete = TRUE</code>). |

Value

A list with:

subbase List of integer vectors. The neighborhoods used as subbase.

base List of integer vectors. The base (closure under intersections).

base_complete Logical. Whether the base closure finished within the `max_base_sets` limit.

connected Logical. Whether the topological space is connected. This is an **exact** result based on the specialization preorder. NA if `check_connected = FALSE`.

components List of integer vectors. The exact topological connected components. Each component is a vector of 1-based element indices.

topology List of integer vectors, or NULL. The open sets, if enumeration was requested and feasible.

n_open_sets Integer, or NA. Number of open sets if enumerated.

complete Logical. Whether the topology enumeration finished within the `max_open_sets` limit.

iterations_base Integer. Wavefront iterations for base closure.

iterations_topo Integer. Wavefront iterations for union closure.

axioms_ok Logical (only if verify_axioms = TRUE and complete = TRUE).

axiom_failure Character (only if axiom verification fails).

References

Nada, S., El Atik, A. E. F., & Atef, M. (2018). New types of topological structures via graphs. *Mathematical Methods in the Applied Sciences*, 41(15), 5801-5810. doi:10.1002/mma.4726

Examples

```
# From a small visibility graph
series <- c(3, 1, 4, 1, 5)
g <- horizontal_visibility_graph(series)
topo <- generate_topology(g$adjacency, g$n)
topo$connected
length(topo$components)
```

horizontal_visibility_graph

Construct the Horizontal Visibility Graph of a time series

Description

Constructs the Horizontal Visibility Graph (HVG) from a time series. Two observations (t_a, y_a) and (t_b, y_b) with $t_a < t_b$ are connected if and only if every intermediate observation (t_c, y_c) satisfies $y_c < \min(y_a, y_b)$.

The HVG is parameter-free and captures structural properties of the series dynamics. It is computed in $O(n)$ time using a stack-based algorithm.

Usage

```
horizontal_visibility_graph(series, directed = FALSE)
```

Arguments

| | |
|----------|--|
| series | Numeric vector representing the time series. The position index is used as the time coordinate. |
| directed | Logical. If TRUE, also returns directed adjacency lists (out_adjacency and in_adjacency) for constructing forward and backward Nada topologies or Alexandrov topologies. Edges are directed from earlier to later time (default: FALSE). |

Details

The algorithm uses a monotone stack with $O(n)$ time and space complexity. Each element is pushed and popped at most once.

The adjacency list returned as `adjacency` provides the neighborhood structure $N(v)$ for each vertex, which serves as the subbase for inducing a topology following Nada et al. (2018).

When `directed = TRUE`, the function additionally returns directed adjacency lists. By construction, all edges satisfy `from < to` (earlier time to later time), so the directed graph is a DAG with the time index as topological order.

Value

A list with:

edges A data.frame with columns `from` and `to` (integer, 1-based indices) representing undirected edges.

n Integer. Number of observations in the series.

n_edges Integer. Number of edges in the graph.

adjacency A list of integer vectors, where element `i` contains the indices of all neighbors of node `i`. This is the neighborhood structure used as subbase for topology induction.

out_adjacency (Only if `directed = TRUE`.) A list of integer vectors, where element `i` contains the indices of all forward (later-in-time) neighbors. Used as subbase for the forward Nada topology τ^+ or as input for the Alexandrov topology.

in_adjacency (Only if `directed = TRUE`.) A list of integer vectors, where element `i` contains the indices of all backward (earlier-in-time) neighbors. Used as subbase for the backward Nada topology τ^- .

References

Luque, B., Lacasa, L., Ballesteros, F., & Luque, J. (2009). Horizontal visibility graphs: exact results for random time series. *Physical Review E*, 80(4), 046103. doi:10.1103/PhysRevE.80.046103

Examples

```
series <- c(3, 1, 4, 1, 5, 9, 2, 6)
g <- horizontal_visibility_graph(series)
g$n_edges
head(g$edges)

# Directed graph for bitopological analysis
gd <- horizontal_visibility_graph(series, directed = TRUE)
gd$out_adjacency[[1]] # forward neighbors of first observation
```

is_topology_connected *Check if a topology is connected using undirected graph approach*

Description

Converts the topology into an undirected graph (two elements share an edge if they appear together in any open set) and checks graph connectivity via depth-first search.

Note: This is a necessary but not sufficient condition for topological connectivity. For an exact check, use [is_topology_connected_exact](#).

Usage

```
is_topology_connected(topology)
```

Arguments

topology A list of integer vectors representing the open sets.

Value

logical scalar. TRUE if the derived graph is connected, FALSE otherwise or if the topology is empty.

Examples

```
topology <- list(c(1, 2, 3), c(3, 4, 5))
is_topology_connected(topology)
```

is_topology_connected_exact
Check topological connectivity of an existing topology

Description

Checks whether a topological space (V, τ) is connected by verifying the exact definition: no proper non-empty open set has an open complement. Uses multi-word bitset representation and hash lookup.

Usage

```
is_topology_connected_exact(topology, n_elements)
```

Arguments

topology A list of integer vectors representing the open sets.
n_elements Integer. Total number of elements in V .

Value

A list with:

connected Logical. TRUE if the space is topologically connected.

component1 Integer vector (only if disconnected). One component.

component2 Integer vector (only if disconnected). The complement.

Examples

```
# A connected topology
tau <- list(integer(0), 1:3, c(1L, 2L), c(2L, 3L))
is_topology_connected_exact(tau, 3L)

# A disconnected topology
tau2 <- list(integer(0), 1:4, c(1L, 2L), c(3L, 4L))
is_topology_connected_exact(tau2, 4L)
```

is_topology_connected_manual

Check if all elements are covered by the topology

Description

Checks whether every integer from 1 to the maximum value in the topology appears in at least one open set. This verifies **coverage** ($\bigcup \tau = X$), not topological connectivity.

Note: A space can have full coverage and be maximally disconnected (e.g., the discrete topology). For connectivity, use [is_topology_connected_exact](#).

Usage

```
is_topology_connected_manual(topology)
```

Arguments

topology A list of integer vectors representing the open sets.

Value

logical scalar. TRUE if all elements from 1 to the maximum are present in at least one set.

Examples

```
topology <- list(c(1, 2, 3), c(3, 4, 5))
is_topology_connected_manual(topology)
```

```
is_topology_connected2
```

Check if a topology is connected using directed graph approach

Description

Converts the topology into a directed graph (sequential edges between sorted elements within each set) and checks reachability via DFS.

Note: This is a necessary but not sufficient condition for topological connectivity. For an exact check, use `is_topology_connected_exact`.

Usage

```
is_topology_connected2(topology)
```

Arguments

`topology` A list of integer vectors representing the open sets.

Value

logical scalar. TRUE if all elements are reachable from the minimum element via directed edges, FALSE otherwise.

Examples

```
topology <- list(c(1, 2, 3), c(3, 4, 5))
is_topology_connected2(topology)
```

```
natural_visibility_graph
```

Construct the Natural Visibility Graph of a time series

Description

Constructs the Natural Visibility Graph (NVG) from a time series. Two observations (t_a, y_a) and (t_b, y_b) with $t_a < t_b$ are connected if and only if every intermediate observation (t_c, y_c) satisfies:

$$y_c < y_a + (y_b - y_a) \cdot \frac{t_c - t_a}{t_b - t_a}$$

The NVG is parameter-free and richer in structure than the HVG (more edges), preserving dynamical properties such as periodicity, chaoticity, and long-range correlations. Computed in $O(n \log n)$ expected time.

Usage

```
natural_visibility_graph(series, directed = FALSE)
```

Arguments

| | |
|----------|--|
| series | Numeric vector representing the time series. The position index is used as the time coordinate. |
| directed | Logical. If TRUE, also returns directed adjacency lists (out_adjacency and in_adjacency) for constructing forward and backward Nada topologies or Alexandrov topologies. Edges are directed from earlier to later time (default: FALSE). |

Details

The algorithm uses divide-and-conquer on the global maximum. The maximum point blocks all cross-edges between the left and right halves, so the problem decomposes cleanly. Edges from the pivot are found by a linear slope scan. Expected time is $O(n \log n)$; worst case (monotone data) is $O(n^2)$.

When directed = TRUE, the function additionally returns directed adjacency lists. By construction, all edges satisfy from < to (earlier time to later time), so the directed graph is a DAG.

Value

A list with the same structure as [horizontal_visibility_graph](#).

References

Lacasa, L., Luque, B., Ballesteros, F., Luque, J., & Nuno, J. C. (2008). From time series to complex networks: The visibility graph. *Proceedings of the National Academy of Sciences*, 105(13), 4972-4975. doi:10.1073/pnas.0709247105

Examples

```
series <- c(3, 1, 4, 1, 5, 9, 2, 6)
g <- natural_visibility_graph(series)
g$n_edges
head(g$edges)

# Directed graph for bitopological analysis
gd <- natural_visibility_graph(series, directed = TRUE)
gd$out_adjacency[[1]] # forward neighbors of first observation
```

`simplest_topology` *Create the discrete topology (completely disconnected)*

Description

Generates the discrete topology on n elements, where the base consists of all singleton sets $\{\{1\}, \{2\}, \dots, \{n\}\}$. The full topology is the power set 2^V , but only the base is returned explicitly (enumerating 2^n sets is impractical for large n).

Usage

```
simplest_topology(data)
```

Arguments

`data` Numeric vector containing the data points.

Value

A list with:

subbase List of singleton sets.

base Same as subbase (singletons are already a base).

topology_type Character: "discrete".

n Integer. Number of elements.

connected Logical. Always FALSE for $n \geq 2$ (the discrete topology is maximally disconnected).

Examples

```
result <- simplest_topology(c(1, 2, 3, 4, 5))
result$connected
```

`visualize_topology_thresholds`
Visualize and compare different threshold methods

Description

Visualize and compare different threshold methods

Usage

```
visualize_topology_thresholds(data, plot = TRUE)
```

Arguments

`data` Numeric vector to analyze.
`plot` Logical indicating whether to return plot objects (default: TRUE).

Value

A `data.frame` with columns `method`, `threshold`, and `base_size`. If `plot = TRUE`, carries an attribute `"plots"` containing a list of three `ggplot` objects.

Examples

```
data <- rnorm(50)
results <- visualize_topology_thresholds(data)
```

Index

`analyze_topology_factors`, [2](#)

`bitopology_invariants`, [3](#), [10](#)

`calculate_thresholds`, [5](#)
`calculate_topology`, [6](#)
`complete_topology`, [6](#)

`generate_alexandrov_topology`, [7](#)
`generate_bitopology`, [4](#), [8](#), [9](#)
`generate_topology`, [2](#), [3](#), [5–9](#), [10](#), [10](#)

`horizontal_visibility_graph`, [7](#), [10](#), [11](#),
[12](#), [17](#)

`is_topology_connected`, [14](#)
`is_topology_connected2`, [16](#)
`is_topology_connected_exact`, [14](#), [14–16](#)
`is_topology_connected_manual`, [15](#)

`natural_visibility_graph`, [7](#), [10](#), [11](#), [16](#)

`simplest_topology`, [18](#)

`visualize_topology_thresholds`, [18](#)